Long Rod Penetrators
Optimum Penetrator Length and Velocity

Basic equation for **cylindric penetrators** ($L_w = L$):

$$P = L \cdot a \cdot \frac{1}{\tanh\left(b_0 + b_1 \cdot \frac{L_w}{D}\right)} \cdot \cos^m \theta \cdot \sqrt{\frac{\rho_P}{\rho_T}} \cdot e^{\frac{-s^2}{v_T^2}} \tag{1}$$

Substitutions for given penetrators and materials:

$$\lambda = \frac{L}{D} \quad \text{or} \quad D = \frac{L}{\lambda} \tag{2}$$

$$k_1 = a \cdot \frac{1}{\tanh(b_0 + b_1 \cdot \lambda)} \cdot \cos^m \theta \cdot \sqrt{\frac{\rho_P}{\rho_T}} \tag{3}$$

Simplified equation:

$$\boxed{P = L \cdot k_1 \cdot e^{\frac{-s^2}{v_T^2}}} \tag{4}$$

Transformation: P as function of the impact energy

Penetrator mass:

$$m_P = \frac{\pi}{4 \cdot \lambda^2} \cdot \rho_P \cdot L^3 \tag{5}$$

Impact energie:

$$E = \frac{m_P \cdot {v_T}^2}{2} = \frac{\pi}{8 \cdot \lambda^2} \cdot \rho_P \cdot L^3 \cdot v_T^2 \tag{6}$$

Rewrite for v_T:

$$v_T^2 = \frac{8 \cdot E \cdot \lambda^2}{\pi \cdot \rho_P \cdot L^3} \tag{7}$$

Insert (7) in (4):

$$\boxed{P = L \cdot k \cdot e^{\frac{-s^2 \cdot \rho_P \cdot \pi \cdot L^3}{8 \cdot E \cdot \lambda^2}}} \tag{8}$$

Substitution:

$$k_2 = \frac{s^2 \cdot \rho_P \cdot \pi}{8 \cdot \lambda^2} \tag{9}$$

$$\boxed{P = L \cdot k_1 \cdot \exp\left(\frac{-k_2 \cdot L^3}{E}\right)} \tag{10}$$

$$\ln\left(\frac{P}{L \cdot k_1}\right) = \frac{-k_2 \cdot L^3}{E}$$

Required energy for perforation P with a given penetrator length L:

$$\boxed{E = \frac{-k_2 \cdot L^3}{\ln\left(\frac{P}{k_1}\right) - \ln(L)}} \tag{11}$$

Optimum penetrator length (minimum energy): $\frac{\delta E}{\delta L} = 0$

$$\frac{\delta E}{\delta L} = \frac{-3 \cdot k_2 \cdot L^2 \cdot \left(\ln\left(\frac{P}{k_1}\right) - \ln(L)\right) + k_2 \cdot L^2}{\left(\ln\left(\frac{p}{k_1}\right) - \ln(L)\right)^2} = 0 \tag{12}$$

Dividend = 0 : $$3 \cdot k_2 \cdot L^2 \cdot \left(\ln\left(\frac{P}{k_1}\right) - \ln(L)\right) = k_2 \cdot L^2$$

$$\ln(L) = \ln\left(\frac{P}{k_1}\right) + \frac{1}{3}$$

Optimum penetrator length for a given perforation length:

$$\boxed{L_{opt} = \frac{P}{k_1} \cdot e^{\frac{1}{3}}} \tag{13}$$

or $$\boxed{L_{opt} = \frac{P \cdot \tanh\left(b_0 + b_1 \cdot \frac{L}{D}\right)}{a \cdot \cos^m \theta \cdot \sqrt{\frac{\rho_P}{\rho_T}}} \cdot e^{\frac{1}{3}}} \tag{14}$$

Energy for opt. penetrator length:

(14) in (11)

$$E=\frac{-k_2\cdot\left(\frac{P}{k_1}\cdot e^{\frac{1}{3}}\right)^3}{\ln\left(\frac{P}{k_1}\right)-\ln\left(\frac{P}{k_1}\cdot e^{\frac{1}{3}}\right)}=\frac{-k_2\cdot e\cdot\left(\frac{P}{k_1}\right)^3}{\ln\left(\frac{P}{k_1}\right)-\ln\left(\frac{P}{k_1}\right)-\frac{1}{3}}=3\cdot k_2\cdot e\cdot\left(\frac{P}{k_1}\right)^3$$

$$\boxed{E=3\cdot k_2\cdot e\cdot\left(\frac{P}{k_1}\right)^3} \qquad (15)$$

k1 and k2 in (15):

$$\boxed{E=\frac{3\cdot s^2\cdot\rho_P\cdot\pi\cdot e}{8\cdot\lambda^2}\cdot\left(\frac{D}{L}\right)^2\cdot\left(\frac{P\cdot\tanh\left(b_0+b_1\cdot\frac{L}{D}\right)}{a\cdot\cos^m\theta\cdot\sqrt{\frac{\rho_P}{\rho_T}}}\right)^3} \qquad (16)$$

Optimum velocity:

(13) and (15) in (7):

$$v_{opt}{}^2=\frac{8\cdot 3\cdot k_2\cdot e\cdot\left(\frac{P}{k_1}\right)^3\cdot\lambda^2}{\pi\cdot\rho_P\cdot\left(e^{\frac{1}{3}}\cdot\frac{P}{k_1}\right)^3}=\frac{\frac{8\cdot 3\cdot s^2\cdot\rho_P\cdot\pi\cdot e\cdot\lambda^2}{8\cdot\lambda^2}\cdot\left(\frac{P}{k_1}\right)^3}{\pi\cdot\rho_P\cdot e\cdot\left(\frac{P}{k_1}\right)^3}=3\cdot s^2$$

$$\boxed{v_{opt}{}^2=3\cdot s^2} \qquad (17)$$

Tungsten and DU penetrators

$$\boxed{v_{opt}=\sqrt{3\cdot\frac{(c_0+c_1\cdot BHNT)\cdot BHNT}{\rho_P}}} \qquad (18)$$

Steel Penetrators

$$\boxed{v_{opt}=\sqrt{3\cdot\frac{c_0\cdot BHNT^k\cdot BHNP^n}{\rho_P}}} \qquad (19)$$